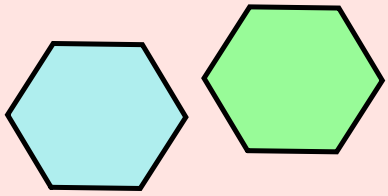
§4.2 Apply Congruence and Triangles

G.CO.7 **Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.**

In earlier lessons you learned about congruent segments and angles.
In this lesson you will develop a definition of congruent polygons.

Polygon Congruence

It is easy to slide a polygon on top of another to see if the polygons are congruent.



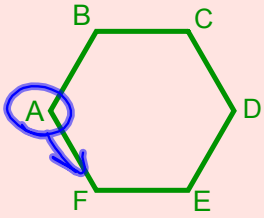
If two polygons are congruent, then their respective angles and sides are congruent.

CONVERSE: If the respective angles and sides of a polygon match, then the polygons are congruent.

NAMING POLYGONS:

Polygons can be named by listing the vertices in order by going around the figure clockwise or counterclockwise.

List all the possible names for the hexagon.



Start with ABCDEF (there are 12 possibilities)

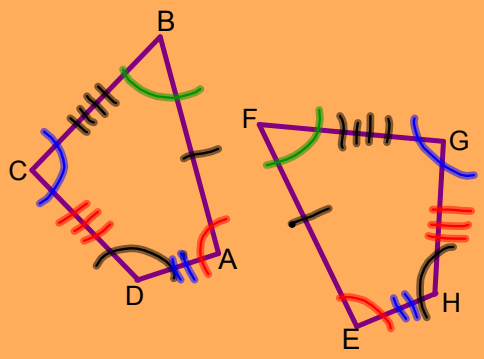
ABCDEF	AFEDCB
BCDEFA	BAFEDC
CDEFAB	CBAFED
DEFABC	DCBAFE
EFABCD	EDCBAF
FABCDE	FEDCBA

ANSWER

Corresponding Sides and Angles

If two polygons have the same number of sides, it is possible to set up a correspondence between them by pairing their parts. In quadrilaterals ABCD and EFGH, for example, you can pair angles

A and E
B and F
C and G
D and H



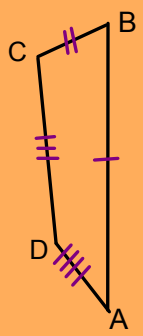
Correspondence of the sides follows from the correspondence of the angles.

Ex.) $\overline{AB}$ corresponds to side $\overline{EF}$ ...

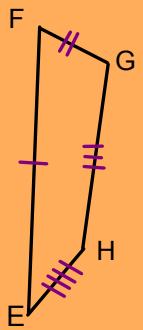
What are others (remember order matters)?

1. $\overline{DA}$ corresponds to side $\overline{HE}$.
2. $\overline{CD}$ corresponds to side $\overline{GH}$.
3. $\overline{BC}$ corresponds to side $\overline{FG}$.

The polygons at the right are congruent. Write ALL congruence statements about them.



All corresponding angles are congruent!



1. ABCD ≅ EFGH
2. BCDA ≅ FGHE
3. CDAB ≅ GHEF
4. DABC ≅ HEFG
5. DCBA ≅ HGFE
6. CBAD ≅ GFEH
7. BADC ≅ FEHG
8. ADCB ≅ EHGF

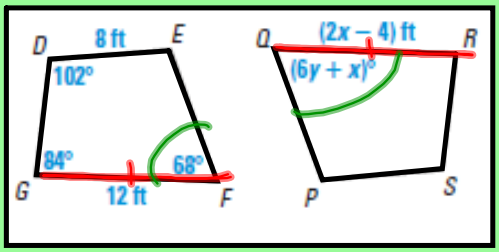
POLYGON CONGRUENCE POSTULATE

Two polygons are congruent iff there is a correspondence between their sides and angles such that:

Each pair of corresponding angles is congruent
Each pair of corresponding sides is congruent

Use properties of congruent figures

In the diagram, $DEFG \cong SPQR$



a. Find the value of *x*.

$2x - 4 = 12$
$+4 \quad +4$
$\frac{2x}{2} = \frac{16}{2}$
$x = 8$

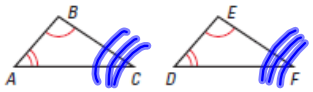
b. Find the value of *y*.

$6y + x = 68$
$6y + 8 = 68$
$-8 \quad -8$
$\frac{6y}{6} = \frac{60}{6}$
$y = 10$

THEOREM *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

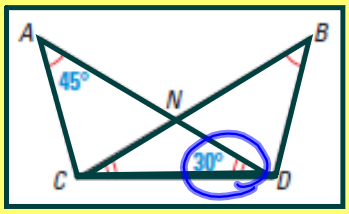
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



If $\angle A = \angle D$, and $\angle B = \angle E$, then $\angle C = \angle F$.

Use the Third Angles Theorem

Find $m\angle BDC$.

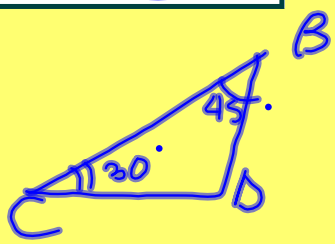


$30 + 45 + x = 180$

$75 + x = 180$

$-75 \qquad -75$

$x = 105°$



Properties of Congruent Triangles. **The properties of congruence that are true for segments and angles are also true for triangles.**

THEOREM *For Your Notebook*

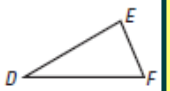
THEOREM 4.4 Properties of Congruent Triangles

Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$

Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$